Equality Constraint Handling for a Surrogate-Assisted Optimizer (SACOBRA)

Samineh Bagheri

Cologne University of Applied Sciences TH Köln
Campus Gummersbach

March 2016
Section 1

SACOBRA
Minimizing mass of a car is one example of real optimization problem. 

- No analytical formulation 
- Parametrization of the problem 
- Determining the constraints by doing the time-expensive crash test simulation 

Optimization problems in industry often are:

- Black-Box 


**What do they want?**

100-500 evaluations
Real World Optimization

Minimizing mass of a car is one example of real optimization problem.

- No analytical formulation
- Parametrization of the problem
- Determining the constraints by doing the time-expensive crash test simulation

Optimization problems in industry often are:

- Black-Box
- High dimensional


What do they want?

100-500 evaluations
Minimizing mass of a car is one example of real optimization problem.

- No analytical formulation
- Parametrization of the problem
- Determining the constraints by doing the time-expensive crash test simulation

Optimization problems in industry often are:

- Black-Box
- High dimensional
- Multi constraints


**What do they want?**

100-500 evaluations
Real World Optimization

Minimizing mass of a car is one example of real optimization problem.

- No analytical formulation
- Parametrization of the problem
- Determining the constraints by doing the time-expensive crash test simulation

Optimization problems in industry often are:

- Black-Box
- High dimensional
- Multi constraints
- Expensive


What do they want?

100-500 evaluations
Minimizing mass of a car is one example of real optimization problem.

- No analytical formulation
- Parametrization of the problem
- Determining the constraints by doing the time-expensive crash test simulation

Optimization problems in industry often are:

- Black-Box
- High dimensional
- Multi constraints
- Expensive


**What do they want?**

100-500 evaluations
State of the art

- COBRA\textsuperscript{1} appeared to be an efficient solver
- Later SACOBRA\textsuperscript{2} which was a modified version of COBRA in R showed competitive results
- These techniques use \textbf{Radial Basis Functions} to model objective and constraint function(s)
- They solve an optimization problem on the surrogates
- Training \textbf{RBF} is fast even in a high dimensional space

\textsuperscript{1}R. Regis, ”Constrained optimization by radial basis function interpolation for high-dimensional expensive black-box problems with infeasible initial points,” Engineering Optimization

\textsuperscript{2}S. Bagheri, W. Konen, T. Bäck and M. Emmerich ”Solving the G-problems in less than 500 iterations: Improved efficient constrained optimization by surrogate modeling and adaptive parameter control”
SACOBRA: Self-Adjusting Constrained Black-Box Optimization with RBF

I. Rescale input space

Generate & evaluate initial design

II. Adjust constraint function(s)

Run repair heuristic

Add solution to the population

III. Adjust DRC

Solution repaired or feasible?

Evaluate new point on real functions

Run optimization on surrogates

IV. Online adjustment of fitness function

Update the best solution

Add solution to the population

Evaluate new point on real functions

V. Select start point (RS)

Fit RBF surrogates of objective and constraints

Budget exhausted?

Yes

No
Section 2

Challenges
Problems with Equality Constraints

Minimize \( f(x) \),
subject to \( g_i(x) \leq 0, \quad i = 1, \ldots, m \)
\( h_j(x) = 0, \quad j = 1, \ldots, r \)

- Problem: equality constraints have zero feasible volume
- SACOBRA was not able to handle equality constraints directly
- Equality-to-inequality transformation does not always work well, \((h_j(x) < 0 \text{ or } h_j(x) > 0)\)

Figure: Examples of optimization problems with equality handling. Green shaded area represents contour plot of the fitness function. Black curve is the equality constraint.
Restricted Equality Constraint

- As long as no feasible solution is found SACOBRA selects the best infeasible solution as the next starting point.
- Solutions like A will be always preferred over B and C.
- The search will get stuck in a wrong area.

\[ f_1 = f(x) \]
\[ f_2 = \sum h_i \]
Section 3

SACOBRA+EH
Shrinking equality margin

$h(x) = 0$
Shrinking equality margin

$h(x) = 0$
Shrinking equality margin

$h(x) = 0$
Repeat the following steps as long as the budget is exhausted

\[ \text{Min} \sum_{j=1}^{r} (s(n_j) \| \vec{x} \|^2) \]

---

3S. Bagheri, W. Konen and T. Bäck "Equality Constraint Handling for Surrogate-Assisted Constrained Optimization" (submitted to WCCI 2016)
Proposed Algorithm

Repeat the following steps as long as the budget is exhausted

- Train new surrogates

---

*S. Bagheri, W. Konen and T. Bäck "Equality Constraint Handling for Surrogate-Assisted Constrained Optimization" (submitted to WCCI 2016)
Proposed Algorithm

Repeat the following steps as long as the budget is exhausted

- Train new surrogates
- Run optimization on surrogates

---

S. Bagheri, W. Konen and T. Bäck "Equality Constraint Handling for Surrogate-Assisted Constrained Optimization" (submitted to WCCI 2016)
Proposed Algorithm

Repeat the following steps as long as the budget is exhausted

- Train new surrogates
- Run optimization on surrogates
- Refine the solution

Refine Step:

$$Min \sum_{j=1}^{r} (s_j^{(n)}(\mathbf{x}))^2$$
Proposed Algorithm

Repeat the following steps as long as the budget is exhausted

- Train new surrogates
- Run optimization on surrogates
- Refine the solution
- Reduce the equality margin

Refine Step:

\[ \text{Min} \; \sum_{j=1}^{r} (s_j^{(n)}(\vec{x}))^2 \]

---

\(^3\)S. Bagheri, W. Konen and T. Bäck "Equality Constraint Handling for Surrogate-Assisted Constrained Optimization" (submitted to WCCI 2016)
Proposed Algorithm\textsuperscript{3}

Repeat the following steps as long as the budget is exhausted

- Train new surrogates
- Run optimization on surrogates
- Refine the solution
- Reduce the equality margin

Refine Step:

$$\text{Min} \quad \sum_{j=1}^{r} (s_{j}^{(n)}(\vec{x}))^2$$

\textsuperscript{3}S. Bagheri, W. Konen and T. Bäck "Equality Constraint Handling for Surrogate-Assisted Constrained Optimization" (submitted to WCCI 2016)
Proposed Algorithm

Repeat the following steps as long as the budget is exhausted

- Train new surrogates
- Run optimization on surrogates
- Refine the solution
- Reduce the equality margin

Refine Step:

\[
\min \sum_{j=1}^{r} (s_j^{(n)}(\bar{x}))^2
\]

---

3S. Bagheri, W. Konen and T. Bäck "Equality Constraint Handling for Surrogate-Assisted Constrained Optimization" (submitted to WCCI 2016)
Proposed Algorithm

Repeat the following steps as long as the budget is exhausted

- Train new surrogates
- Run optimization on surrogates
- Refine the solution
- Reduce the equality margin

Refine Step:

\[
\text{Min } \sum_{j=1}^{r} (s_j^{(n)}(\vec{x}))^2
\]

---

S. Bagheri, W. Konen and T. Bäck "Equality Constraint Handling for Surrogate-Assisted Constrained Optimization" (submitted to WCCI 2016)
### G function Suite

**Figure:** Characteristic of G-functions

<table>
<thead>
<tr>
<th>Fct.</th>
<th>d</th>
<th>type</th>
<th>LI</th>
<th>NI</th>
<th>LE</th>
<th>NE</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>G03</td>
<td>20</td>
<td>nonlinear</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>G05</td>
<td>4</td>
<td>nonlinear</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>G11</td>
<td>2</td>
<td>nonlinear</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>G13</td>
<td>5</td>
<td>quadratic</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>G14</td>
<td>10</td>
<td>nonlinear</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>G15</td>
<td>3</td>
<td>quadratic</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>G17</td>
<td>6</td>
<td>nonlinear</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>G21</td>
<td>7</td>
<td>nonlinear</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

- The well-studied G function suite contains 24 constrained optimization problems
- We use 8 problems which have equality constraints
- Real-world problems, artificial problems
- They are diverse and challenging. There is no single algorithm which can handle all of these problems
Results

\begin{center}
\begin{figure}[h]
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{G03.png}
\caption{G03 problem}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{G05.png}
\caption{G05 problem}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{G11.png}
\caption{G11 problem}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{G13.png}
\caption{G13 problem}
\end{subfigure}
\end{figure}
\end{center}
Compare with the state-of-the-art

<table>
<thead>
<tr>
<th>Fct.</th>
<th>Optimum</th>
<th>SACOBRA+EH [this work]</th>
<th>Zhang</th>
<th>ISRES</th>
<th>RGA 10%</th>
<th>Jiao</th>
<th>DE</th>
</tr>
</thead>
<tbody>
<tr>
<td>G03</td>
<td>-1.0</td>
<td>-1.0</td>
<td>-1.001</td>
<td>-0.9999</td>
<td>-1.0005</td>
<td>-0.8414</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>m 100</td>
<td>25493</td>
<td>349200</td>
<td>399804</td>
<td>19534</td>
<td>13325</td>
</tr>
<tr>
<td>G05</td>
<td>5126.497</td>
<td>5126.497</td>
<td>5126.497</td>
<td>5126.498</td>
<td>5126.497</td>
<td>5126.498</td>
<td>5126.498</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m 300</td>
<td>21363</td>
<td>195600</td>
<td>39459</td>
<td>2050</td>
<td>8108</td>
</tr>
<tr>
<td>G11</td>
<td>0.750</td>
<td>0.750</td>
<td>0.749</td>
<td>0.750</td>
<td>0.750</td>
<td>0.749</td>
<td>0.750</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m 100</td>
<td>6609</td>
<td>137200</td>
<td>7215</td>
<td>135</td>
<td>2099</td>
</tr>
<tr>
<td>G13</td>
<td>0.0539</td>
<td>0.0539</td>
<td>0.0539</td>
<td>0.0539</td>
<td>–</td>
<td>0.0539</td>
<td>0.068</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m 300</td>
<td>19180</td>
<td>223600</td>
<td>–</td>
<td>3103</td>
<td>23637</td>
</tr>
<tr>
<td>G14</td>
<td>-47.765</td>
<td>-47.765</td>
<td>-47.765</td>
<td>–</td>
<td>–</td>
<td>-47.765</td>
<td>-47.761</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m 500</td>
<td>34825</td>
<td>–</td>
<td>–</td>
<td>6093</td>
<td>72015</td>
</tr>
<tr>
<td>G15</td>
<td>961.715</td>
<td>961.715</td>
<td>961.715</td>
<td>–</td>
<td>–</td>
<td>961.715</td>
<td>961.715</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m 500</td>
<td>11706</td>
<td>–</td>
<td>–</td>
<td>757</td>
<td>5666</td>
</tr>
<tr>
<td>G17</td>
<td>8853.534</td>
<td>8853.794</td>
<td>8868.539</td>
<td>–</td>
<td>–</td>
<td>8853.534</td>
<td>8867.606</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m 500</td>
<td>43369</td>
<td>–</td>
<td>–</td>
<td>3203</td>
<td>37532</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m 500</td>
<td>23631</td>
<td>–</td>
<td>–</td>
<td>46722</td>
<td>35559</td>
</tr>
<tr>
<td></td>
<td></td>
<td>average fe</td>
<td>350</td>
<td>23272</td>
<td>226400</td>
<td>148826</td>
<td>10200</td>
</tr>
</tbody>
</table>
Section 4

Conclusion
Conclusion

- Problems with equality constraints are challenging
- We have proposed an effective approach to handle equality constraints
- SACOBRA+EH solves all the listed problems with less than 500 iterations
Thank You For Your Attention